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ON THE GENERATION MECHANISM OF ELECTROMAGNETIC VLF RADIATION IN THE OUTER VAN ALLEN BELT

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ABSTRACT

It is shown in this paper that the distribution of electrons by velocities in the outer radiation belt (Van Allen belt) may be unsteady relative to low-frequency electromagnetic oscillations. This instability is the basis for explaining the origin and a series of peculiarities of VLF-radiation in the Earth's exosphere.

* * *

A large experimental material has recently been accumulated in regard to natural VLF-radiation [1, 2]. There are two types of VLF - the discrete and the continuous radiation (emission). A series of works were devoted to explaining the discrete VLF-signals [1, 3]. We shall only consider the continuous type of VLF. This radiation usually ranges in the $1 \div 30$ kc/s frequency band with a maximum intensity in the frequency of ~ 5 kc/s, but considerably higher frequencies were received (to 200 kc/s) [4]. The radionoise mostly lasts $1.5 \div 2$, and sometimes lasts beyond 10 hours.

The VLF-radiation is received in a broad geomagnetic latitude range. According to latest data the least reception latitude is $35.7°$ [5]. The angular dimensions of the emission source by azimuth are

* O mekhnizme generatsii elektromagnitnogo ul'tranizkochastotnogo izlucheniya vo vneshnem radiatsionnom poyase Zemli.

usually great, reaching 50^{o} and more [6]. The intensity of VLF-signals received by ground stations is very low ($0.05 \div 0.5$ $mvm^{-1} \cdot cps$), with a maximum at about 60^{o} latitude [6, 7]. The VLF reveals a notable correlation with geomagnetic activity. The most frequently received hiss has the tendency of occurring at time of reduced phase of a geomagnetic storm [1].

There are indications of existence of relationship between the VLF-radiation and the occurrence of aurorae [8, 9].

A series of mechanisms have been proposed for the explanation of VLF-radiation [1, 3, 10 — 12]. They are based on Čerenkov radiation and brehmstrahlung of cloud or charged particle flux, moving through the Earth's exosphere. While these mechanisms explain satisfactorily the origin and the peculiarities of discrete-type emission, they are inapplicable for the description of a series of peculiarities of VLF-noises, having a long duration and a broad stationary frequency band. Thus if we have recourse to the incoherent particle emission mechanism in the ionosphere and exosphere for the explanation of the observed continuous VLF-noise, the radiative power obtained according to calculations of [13], results by several orders lower than that observed. For example, the incoherent Čerenkov emission of highly-energetic electrons from the outer radiation belt assures a power of 10^{-19} watt m^{-2} cps^{-1} in the frequency of 5 kc/s. The contribution from the incoherent synchrotron emission of radiation belts' particles results still lower [13].

Converging values are obtained when estimating the intensity of the incoherent emission from a stationary solar corpuscular flux. Thus, taking the stream's velocity as being $v_c \sim 3 \cdot 10^{8}$ cm/sec, the particle concentration in the flux — as 10^{2} cm^{-3} and the characteristic dimension of the stream as $\sim 10^{9}$ cm, and utilizing the expressions of [13], we obtain for radiation (emission) intensity a quantity $\sim 3 \cdot 10^{-20}$ watt m^{-2} cps^{-1} in the frequency $f \sim 5$ kc/s. But the observed intensities constitute for example at 9 kc/s frequency a value $\sim 10^{-15}$ watt m^{-2} cps^{-1} [4].

The coherent emission of the flux of charged particles could in principle assure an intensity by several orders greater than the incoherent. But this fails to explain a whole series of peculiarities characteristic of VLF. Thus, assuming for example a VLF generation by a solar corpuscular stream, it is impossible to eliminate the contradiction consisting in that the emission often correlates with the reduced phase of a magnetic storm [1], when there is no flux. It is difficult to explain the intensity maximum at 5 kc/s and the well-expressed stationary state of the radionoise.

Therefore, the existence of a continuous VLF- component endowed with the above-described properties, cannot be explained by radiation generation mechanism originating directly in solar corpuscular streams penetrating the Earth's exosphere.

We propose below a mechanism of VLF-noise generation in the outer Van Allen belt. Its examination shows that the distribution function of radiation belt's electrons by velocities may be unsteady relative to low-frequency electromagnetic oscillations. This instability is being used for the explanation of the origin and of the series of peculiarities of the continuous VLF-radiation in the Earth's upper atmosphere.

ON THE INSTABILITY OF ELECTRONS'DISTRIBUTION FUNCTION IN THE OUTER RADIATION BELT.

Let us consider the distribution function in a plasma situated in a nonuniform magnetic field, In this case, as shown by Parker [14], neglecting the collisions, the charged particles' distribution function $F(\mathbf{p}, \mathbf{r})$ for an equilibrium state $(\partial/\partial t \approx 0)$ may be represented in the form

$$F(\mathbf{p}, \mathbf{r})\, d\mathbf{p} = f(p^2)\, \Phi(\varphi, \theta)\, dp d\theta, \qquad (1)$$

where $f(p^2)$ is the particles' distribution function by energies (p^2 is the square of particle's pulse). $\Phi(\varphi, \theta)$ is the dependence of particle density on the coordinate φ about a selected line of force and on the value of the angle θ between the pulse vector of the particle **p** and the direction of the magnetic field H. A spherical system of coordinates was chosen in the pulses' space.

Parker [14] obtained a general expression for

$$\Phi(\varphi, \theta) = \left[\frac{H(\varphi)}{H(0)}\right]^{1/2} \int d\gamma' C(\gamma') \left[\frac{H(0)}{H(\varphi)} \sin^2\theta\right]^{\frac{\gamma'+1}{2}}$$

Let us consider the special case when $C(\gamma') = C_1 \delta(\gamma' - \gamma)$, agreeing well with the experimental data of [15]. Then

$$\Phi(\varphi, \theta) = C_1 \sin^{\gamma+1}\theta \left[\frac{H(0)}{H(\varphi)}\right]^{\gamma/2}, \qquad (2)$$

where C_1 is the normalization constant; H(0) is the magnetic field at a certain selected point. The case $\gamma = 0$ responds to the isotropic distribution by pulses. At the same time the particle density is constant along the selected line of force of the magnetic field. If $\gamma > 0$, $P_\perp > P_\parallel$ ($P_\perp$ is a component of the total kinetic plasma pressure across the magnetic field **H**, $P_\parallel$ is the longitudinal component of that pressure) and particle concentration decreases as the magnetic field increases; if $\gamma < 0$, $P_\perp < P_\parallel$ and the concentration increases with the increase of H.

We shall utilize the function (1), where Φ is determined by the expression (2), to describe the distribution of electrons in the outer radiatiin belt of the Earth. As the estimates show, the role of collisions between particles of this belt is insignificant.

Since the energy spectrum of particles (electrons) of the belt is at present not sufficiently well knownm we shall use for concrete computations, for $f(p^2)$ the normal distribution*

$$f(p^2) = \frac{\exp\left\{-\frac{p^2}{2mT}\right\}}{(2\pi mT)^{3/2}} p^2, \qquad (3)$$

where m is the particle's mass, T is the temperature in energy units. Then, according to (3), the distribution function of electrons in the outer radiation belt may be written in the form

$$Fdp = (F_1 + F_2)\,dp\,d\theta = \left\{ \frac{\exp\left(-\frac{p^2}{2mT_0}\right)}{(2\pi mT_0)^{3/2}} p^2 \sin\theta + \right.$$
$$\left. + \frac{N_n}{N_0} \frac{\exp\left(-\frac{p^2}{2mT_1}\right)}{(2\pi mT_1)^{3/2}} p^2 \sin^{\gamma+1}\theta \left[\frac{H(0)}{H(\varphi)}\right]^{\gamma/2} \right\} dp\,d\theta. \qquad (4)$$
$$(T_0 \sim 0.25\ \text{ev}).$$

where F_1 is the isotropic Maxwellian distribution of particles of the exosphere proper; F_2 is the distribution of belt's high-energy electrons; N_n and N_0 are respectively the concentrations of particles in the belt and in the exosphere; φ is the geomagnetic latitude; $H(0)$ is the magnetic field in the equatorial part of the radiation belt.

We shall investigate the function (4) for instability relative to electromagnetic oscillations. It is easy to see that the spatial variation of the distribution function can be neglected provided the following condition is fulfilled :

$$\delta \equiv \left|\frac{\lambda}{zF_2}\frac{\partial F_2}{\partial\varphi}\right| \ll 1 \qquad (\lambda = 2\pi\lambda),$$

where z is the characteristic dimension of the belt along $\mathbf{H}$; is the wavelength in the medium. We must note that in all practically interesting cases $\delta < 10^{-3}$ under Earth's exosphere conditions, where the magnetic field is described by a dipole field [2], and for the considered wavelengths.

We shall make use for the analysis of function (4) for instability by the general correlations brought out in the work [16]. We shall write the dispersion equation of normal-type wave propagation

$$Q(\omega;\ \mathbf{k}) - \eta(\omega, \mathbf{k}) = 0. \qquad (5)$$

Here the first addend represents the dispersion equation of electromagnetic waves in a uniform magnetoactive plasma without taking into account the thermal scattering of electrons by velocities

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$$Q = (1 - u - v + uv\cos^2\alpha)\,n^4 - [2(1-v)^2 + uv\cos^2\alpha - u(2-v)]\,n^2 + (1-v)[(1-v)^2 - u], \tag{6}$$

$$u = \frac{\omega_H^2}{\omega^2},\; v = \frac{\omega_0^2}{\omega^2},\; \omega_H = \frac{eH}{mc},\; \omega_0 = \left(\frac{4\pi e^2 N}{m}\right)^{1/2}$$

where n is the index of refraction; e and m are the charge and the mass of the electron; α is the angle between the wave vector **k** and **H**.

Assuming in (6) $v \gg u > 1$, (These inequalities are well satisfied in the exosphere for the radiation of interest to us, where $\omega < \omega_H$), we obtain the following expression for

$$n^2 = v(\sqrt{u}\cos\alpha - 1)^{-1}. \tag{7}$$

The propagation of waves is fully determined by the cold plasma component, i.e. by the exosphere proper, inasmuch as the radiation belt's particles usually constitute no more than 1% of the total number of particles.

The second term $\eta(\omega, k)$ determines the contribution from the accounting of the thermal scattering of particles by velocities. In the approximation

$$\left|\frac{\partial Q}{\partial k}\right| \gg \left|\frac{\eta}{k}\right|,\; \left|\frac{\partial \eta}{\partial k}\right|$$

we shall obtain for the damping (accretion) factor q the following expression:

$$q = \mathrm{Im}\, k = \mathrm{Im}\,\eta \left(\frac{\partial Q}{\partial k}\right)^{-1}. \tag{8}$$

In the correlation (8) the wave vector **k** is the radical of the equation Q = 0. According to [16],

$$\mathrm{Im}\,\eta = \pi(u-1)\sum_{s,j}\int dp_\perp \left[\frac{M_s(\omega, \mathbf{k}, p_\parallel, p_\perp)}{k_z}\right]_{p_\parallel = p_{\parallel s,j}}. \tag{9}$$

The quantity $p_{\parallel s,j}$ is the true radical of the expression

$$\omega m - k_z p_\parallel - sm\omega_{nj} = 0. \tag{10}$$

The summing up in (1)) is effected by all sorts of partcles j and harmonics s. The quantity M_s is expressed by a well known means the Bessel functions and the first derivatives from the distribution function (4) by the lomgitudinal and transeverse pulses $p_{\parallel}$ and $p_{\perp}$; k_z is the wave vector component along the magnetic field H.

As in (7), we shall omit the motion of ions (this corresponds to the case $\omega \gg \Omega_H$, where Ω is the ion gyrofrequency). In the assumption of smallness of Bessel functions' arguments entering in M_s, i.e. at the fulfillment of the correlation [16]

$$(k_z p_{\perp} / m\omega_H)^2 \ll 1 \tag{11}$$

the expression (9) will be reduced to the sum of three integrals

$$\mathrm{Im}\,\eta = \pi (u - 1) [I_1 + I_{-1} + I_0], \tag{12}$$

which will in their turn be determined by the following correlations:

$$I_{\pm 1} = - \frac{\pi v^3 [\mp \sin^2 \alpha + \sqrt{u} (1 \pm \cos \alpha)^2]}{2 (\sqrt{u} \pm 1)(\sqrt{u} \cos \alpha - 1) k_z} \int_0^{\infty} dp_{\perp} p_{\perp}^3 A_{\pm 1}, \tag{13}$$

$$I_0 = - \frac{2 \pi v^3 \sqrt{u} \cos \alpha \sin^2 \alpha \omega^2 m^2}{(u - 1)(\sqrt{u} \cos \alpha - 1)^2 k_z^3} \int_0^{\infty} dp_{\perp} p_{\perp} A_0,$$

where

$$A_{\pm 1} = m \left[\omega \frac{\partial F}{\partial p_{\parallel}^2} \pm \omega_H \left(\frac{\partial F}{\partial p_{\perp}^2} - \frac{\partial F}{\partial p_{\parallel}^2} \right) \right]_{p_{\parallel} = a_{\pm 1} = \frac{\omega \mp \omega_H}{k_z} m}, \tag{14}$$

$$A_0 = \left[m\omega \frac{\partial F}{\partial_{\perp}^2} \right]_{p_{\parallel} = a_0 = \frac{\omega}{k_z} m} p.$$

The expressions (13) are obtained in the assumption

$$\left| A_{\pm 1,0} - 2 m\omega p_{\perp} \frac{\partial F}{\partial p_{\perp}^2} \right| \sin^2 \alpha \ll | A_{\pm 1,0} | \frac{v}{n^2},$$

$$\left| A_0 \frac{k_z p_{\perp}}{m\omega_H} \right| \sin^2 \alpha \ll | A_{\pm 1,0} | (\sqrt{u} - 1). \tag{15}$$

At A_1 not near zero, conditions (15) as reduced to one

$$\frac{\sin^2 \alpha}{\sqrt{u} \cos \alpha - 1} \ll 1,$$

which is upset only in the narrow region of angles adjacent to

It is not difficult to satisfy oneself that inasmuch as the temperature of the particles of the outer radiation belt exceeds by several orders that of the surrounding plasma, the exosphere contributes insignificantly to wave damping (accretion). In this case, substituting in (13) $A_{\pm 1}$ by their values found by utilization of (4), we shall have

$$I_{\pm 1} = \frac{K_{\pm 1} e^{-\frac{a_{\pm 1}^2}{2mT_1}}}{(2\pi m T_1)^{3/2}} \frac{N_n^m}{N_0} \left\{ \int_0^\infty dp_\perp p_\perp^3 \left(\frac{p_\perp^2}{p^2 + a_{\pm 1}^2} \right)^{\gamma/2} \left[\frac{\omega}{2T_1 m} \quad \frac{\gamma}{2} \frac{\omega}{p_\perp^2 + a_{\pm 1}^2} \pm \right.\right.$$

$$\left.\left. + \frac{\gamma \omega_H}{2p^2} \right] e^{-\frac{p_\perp^2}{2mT_1}} \right\}, \tag{16}$$

$$I_0 = \frac{K_0 e^{-\frac{a_0^2}{2mT_1}}}{(2\pi m T_1)^{3/2}} \frac{N_n^m}{N_0} \left\{ \int_0^\infty dp_\perp p_\perp \left[\frac{1}{2T_1 m} + \frac{\gamma}{2(p_\perp^2 + a_0^2)} \right] \left(\frac{p_\perp^2}{p_\perp^2 + a_0^2} \right)^{\gamma/2} e^{-\frac{p_\perp^2}{2mT_1}} \right\}.$$

The respective expressions, standing before the integrals in the correlation (13) are the coefficients $K_{\pm 1.0}$. As earlier

$$a_{\pm 1} = \frac{\omega \mp \omega_H}{k_z} m, \quad a_0 = \frac{\omega}{k_z} m.$$

Let us consider two boundary cases: $a_{\pm 1,0}^2 \gg p_\perp^2$ and $a_{\pm 1,0} \ll p_\perp^2$. In the first case*, when

$$a_{\pm 1,0} \gg p_\perp^2, \tag{17}$$

the expressions (16) for a whole γ take the form:

$$I_{\pm 1} = \frac{K_{\pm 1}}{L v_T} \frac{N_n}{N_0} \left(\frac{mT_1}{a_{\pm 1}^2} \right)^{\frac{\gamma}{2}} (\gamma + 2)!\, e^{-\frac{a_{\pm 1}^2}{2mT_1}} \left(\omega \mp \frac{\gamma}{\gamma + 2} \omega_H \right),$$

$$I_0 = \frac{K_0 \gamma! \omega}{(8\pi m T_1) v_T} \left(\frac{mT}{a_0^2} \right)^{\gamma/2} \frac{N_n}{N_0} e^{-\frac{a_0^2}{2mT_1}}$$

$$\left(\gamma = -1, 0, 1, 2, \ldots, \quad v_T = \sqrt{\frac{T_1}{m}}, \; L = \begin{cases} (2\pi)^{3/2} & \text{at } \gamma = 2n \\ 4\pi & \text{at } \gamma = 2n - 1,\; n = 0, 1, 2, \ldots \end{cases} \right)$$

In the other boundary case, when

$$a_{\pm 1,0} \ll p_{\perp}^2, \tag{18}$$

we shall obtain for $I_{\pm 1.0}$ at arbitrary value of , the following expressions:

$$I_{\pm 1} = \frac{K_{\pm 1} e^{-\frac{a_{\pm 1}^2}{2mT_1}}}{\pi (2\pi)^{1/2} v_{T1}} \frac{N_n}{N_0} \left[\omega + \frac{\gamma}{2} (\omega \mp \omega_H) \right],$$

$$I_0 = \frac{K_0 e^{-\frac{a_0^2}{2mT_1}}}{(2\pi m T_1)^{1/2}} \frac{N_n}{N_0} \left[1 + \frac{\gamma}{2} \ln \frac{2mT_1}{a_0^2} \right] m\omega. \tag{19}$$

Let us consider the case of longitudinal distribution, when $\mathbf{k} \parallel \mathbf{H}$, but at arbitrary direction of the wave vector we shall limit ourselves mainly to a qualitative analysis of the quantities of interest to us. If $\alpha = 0$, I_{-1} and I_0, characterizing at $\gamma > 0$ the specific wave damping linked with the Cerenkov and brehmstrahlung absorption in the anomalous Doppler effect-region, disappear, and the expression for the factor of q amplification takes a simple form:

$$q = \frac{\pi^2}{2L} \left(\frac{n\beta_{T_1}}{\sqrt{\bar{u}} - 1} \right)^{\gamma - 1} \frac{N_n}{N_0} k (\gamma + 2)!\, e^{-\frac{(\sqrt{\bar{u}} - 1)^2}{2n^2 \beta_{T_1}^2}} \left(\frac{\gamma}{\gamma + 2} \sqrt{\bar{u}} - 1 \right) \quad (\beta_{T_1} = v_{T1}/c). \tag{20}$$

Formula (20) is valid when the inequality (17) is fulfilled. In the of the inverse inequality fulfillmnet (18) for q at longitudinal propagation, we shall have the following expression:

$$q = \frac{\sqrt{\bar{u}} - 1}{v_T} \sqrt{\frac{\pi}{8}} \frac{N_n}{N_0} \left[\frac{\gamma}{2} \omega_H - \omega \left(1 + \frac{\gamma}{2} \right) \right]. \tag{21}$$

Formulae (20) and (21) pass at $\gamma = 0$ into the well known expression for the brehmstrahlung absorption coefficient for ordinary waves [17]. It must be pointed out that the condition (11) limits the applicability of formulae (19) and (21), obtained at fulfillment of the inequality (18), to the region of small angles α

$$\alpha < \alpha_{\kappa p} = \arcsin \frac{\omega_H}{k v_\perp} < \frac{\pi}{2}$$

As may be seen from expressions (20) and (21), instability is possible in both cases, determined by the expressions (17) and (18), at frequencies

$$\omega < \omega_{rp} = \frac{\gamma\omega_H}{\gamma+2} \qquad \gamma > 0. \tag{22}$$

The physical nature of this instability is the same as the instability of the distribution function with anisotropic temperature in case $T_\perp > T_\parallel$ [18, 19]. If the mean thermal velocity of radiation belt's particles is sufficiently great. so that condition (18) is satisfied, then, as follows from (21), the amplification factor of **q** rises with the decrease in frequency. In the case when **q** is determined by the expression (20), the maximum radiation can be found from the square equation $(\omega \ll \omega_H)$

$$\frac{\partial q}{\partial \omega} = \omega^2 + \omega\left[\frac{\omega_H^3}{\gamma\omega_0^2\beta_{T_1}^2} - \omega_H\frac{\gamma-2}{\gamma+2}\right] - \frac{\omega_H^4}{(\gamma+2)\,\omega_0^2\beta_T^2} = 0. \tag{23}$$

COMPARISON WITH THE OBSERVATION DATA

The recent observations by means of artifical Earth' satellites and rockets have revealed a considerable irregularity in the distribution of electrons about the magnetic lines of force at heights corresponding to the outer radiation belt. According to the expression (2), this is evidence of the presence of a clearly expressed anisotropy and electron distribution by angles θ. Analysis conducted in reference [15] shows that the anisotropic factor, correponding to unperturbed periods in the outer belt region usually is $\gamma \approx 1$, increasing to $\gamma = 2$ during magnetic storms.

For concrete computations similar to those in [20], we shall admit $\gamma = 1$. Let the intensity maximum of the radiation belt in the equatorial region be remote from the Earth's center by 3.5 Earth's radii [20, 21]. Then, according to (17), the boundary frequency will be

$\omega_{lim} = \omega_H/3$ which corresponds to the range of generated frequencies, from 500 kc to about hundreds of cps.

We shall assume the mean kinetic energy of belt partcles to be 10 kev [20, 21]. For the given exosphere and radiation belt's parameters, we can compute the relation

$$\left(\frac{a_1}{p_\perp}\right)^2 \sim \frac{\omega_H^3}{\omega_0^2\omega\beta_{T_1}^2} \qquad (\omega \ll \omega_H). \tag{24}$$

The values of this relation are compiled below. As may be seen, it is greater than the unity in the frequency $\omega < \omega_{lim}$.

$R = r/r_0$	4	3,5	3	2	1,17
$(a_1/p_\perp)^2_{\omega=\omega_{lim}}$	1,5	3,17	2,7	2,5	9.

r being the distance from the center of the Earth to the generation point.

When computing (24) for the heights h = 1000 ÷ 13 000 km, we took the exponential distribution of electrons [22]

$$N_0 = 3{,}75\cdot 10^4 \exp\left\{-\frac{h}{2640}\right\} \text{см}^{-3}.$$

Above 13 000 km the concentration was estimated constant and equal to 10^2 cm^{-3}.

Since the condition (17) is satisfied, we may use all formulae obtained for that case. The maximum emission frequency will be determined by the following expression obtained from (23):

$$\omega_{max} = \omega_H\left(3 + \frac{\omega_0^2\beta_{T_1}^2}{\omega_H^2}\right)^{-1}. \tag{25}$$

For a more intense part of the radiation belt we shall have

$$f_{max}(r = 3{,}5r_0) = \frac{\omega_{max}}{2\pi} = 4{,}8 \text{ кгц}. \tag{26}$$

This value agrees well with the experimental data. As is shown by observations of [1, 8, 9], the maximum emission frequency usually oscillates between 3 and 9 kc/s. increasing during magnetic storms.

It is possible that these oscillations are related to spatial shift of the radiation belt [21]. Thus, at radiation belt's shift by $\pm\frac{1}{2}\, r_0$ from $r = 3.5\, r_0$ the maximum frequency f_{max} varies by about ∓ 2.4 kc/s, becoming equal to either 2.5 or 7.2 kc/s. Besides, f_{max} increases somewhat with γ rise. At the same time, the intensity of VLF-radiation increases.

As it should have been expected, the spatial displacement of the belt toward the Earth during geomagnetic storms is attended by a shift of the maximum of VLF-radiation occurrence toward lower latitudes. Thus, if in quiet days the probability of VLF-occurrence is maximum at 60° latitude, the low-frequency radionoise will occur more often near $\varphi \sim 55^{\circ}$ in time of magnetic storms [7].

Let us consider the question as to what is the correlation of radiation powers in various frequencies. In case $\gamma = 1$ and utilizing (20) and (25), we obtain for q_{max} with $(\omega \ll \omega_H)$ the following expression:

$$q_{max} = \frac{\pi}{4} \frac{\omega_0^2 \beta_{T_1}^2}{\omega_H^2} \frac{N_n}{N_0} k_{max} \exp\left\{-\frac{\omega_H^2}{\omega_0^2 \beta_{T_1}^2} \frac{\omega_H}{\omega_{max}}\right\}. \qquad (25a)$$

The comparative intensity of the received radiation in two different frequencies may be approximately estimated by amplitude squares' ratio after wave's passing the effective amplification path over the extent of which the amplification factor drops by e times*.

$$\frac{I_1}{I_2} = \frac{E_1^2}{E_2^2} \approx \exp\{2\,[q_{max}(\omega_1)\, z_{ef_1} - q_{max}(\omega_2)\, z_{ef_2}]\}. \qquad (27)$$

Let us appraise the relation (27) for $f_1 = 9$ and $f_2 = 230$ kc/s. Substituting the numerical values of the quantities entering in (25a), (f_1 and f_2 are respectively generated at $h_1 = 11\,300$ km and $h \approx 2000$ km), we obtain the following value for q_{max} in frequency f_1:

$$q_{1\,max} \approx 7.5 \; 10^{-9} N_n \text{ cm}^{-1}.$$

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* For a more precise estimate it is necessary to account for nonlinear effects, leading to settling of oscillations' amplitude.

At the same time, the ratio $q_{1max}/q_{2max} \gg 1$. The quantity z_{ef} is found from the correlation

$$\Delta \frac{a_1^2}{2mT_1} = \frac{1}{\beta_{T_1}^2 \omega_{i max}} \left[\left(\frac{\omega_{H2}^3}{\omega_{02}^2} \right)_{q=\frac{q_{max}}{l}} - \left(\frac{\omega_{H1}^3}{\omega_{01}^2} \right)_{q=q_{max}} \right] = 1.$$

For $f_1 = 9$ kc/s, the value $z_{eff} = 1500$ km. Assuming the density of electrons in the radiation belt at the place of frequency f_1 generation to be $N_n \sim 3 \div 5$ cm^{-3} [21], we shall obtain for the relation (27) the following value:

$$\frac{I_1}{I_2} \approx e^{2.3N_n} \sim 10^4 \div 10^5. \tag{28}$$

This result agrees well with the observation data of [4], according to which $I_1 / I_2 \sim 3 \cdot 10^3$.

As follows from (28), the radiation intensity is very sensitive to variations of particle concentration in the belt. That is why the VLF-noise, which probably is always present, must increase by many times during the periods, when particle density in the radiation belt is maximum. As follows from [6], this corresponds to periods of aurorae and reduced phases of magnetic storms. The increased VLF activity was precisely observed at that time. (see [1, 8, 9]). Another cause leading to the rise in radiation intensity may be the increase of γ during geomagnetic storm periods.

VLF-radiation can serve as an effective mechanism for creating particles responsible for the mid-latitude aurorae [23], inasmuch as in the considered type of instability the emitting particle passes to the state with a lesser angle θ [18], resulting in the increase of particle flow into the "forbidden cone", from which they hit the lower part of the ionosphere and produce aurorae. Mid-latitude aurorae are always attended by the VLF-noise [8], and they line up in the $\Delta\varphi \sim 52 \div 58^{\circ}$* geomagnetic latitude range [8], which agrees well with the position of the outer radiation belt.

* (see ref. [24]).

From the viewpoint of the above-developed theory the fact of the existence of multiple atmospheric echoes and of discrete types of VLF-radiation is of interest [1, 25]. In certain cases, the subsequent echoes were stronger than the preceding ones. The possible explanation of this phenomena may be the fact of whistler and discrete VLF-signal amplification in the outer radiation belt. The fact that in whistler spectra frequencies $f \sim 3 \div 4$ kc/s are heard during longer time speaks in favor of this hypothesis [25], as they are near f_{max}. The multiple echoes must be more frequently observed at the latitude corresponding to whistlers hitting the outer belt's intensity maximum. This latitude is unilaterally determined by the position of the belt and the pattern of whistle propagation trajectory. The study of the distribution of intensity of multiple whistler echoes by latitude at a known spatial position of the outer radiation belt should allow to resolve the old problem of the shape of trajectory of whistler propagation.

In case of arbitrary angle between **k** and **H** an additional damping occurs, which is linked with the Cerenkov and brehmstrahlung absorption in the anomalous Doppler effect region (the terms I_0 and I_{-1} in the expression (12)). The amplification factor then decreases with the increase of α for two reasons: because of the decrease of I_{+1} and of increase of I_{-1} and I_0. As is shown by the esyimates, the influence of the terms I_{-1} and I_0 because essential only for sufficiently great $\alpha \gtrsim 60^{\circ}$. The amplification factor drops rapidly in this region and the maximum emission frequency decreases also.

If the mean kinetic energy of belt's W particles is different from 10 kev, the above-conducted examination remains valid at $W \leqslant 30$ kev (the inequality (17) is satisfied). In case $W > 30$ kev, the inverse inequality (18) is fulfilled and the amplification factor increases with frequency decrease. The maximum value of q will be in the $\alpha_1 \geqslant \overline{\rho}_1$ region.

Thus, the obtained conclusion for the maximum radiation are qualitatively applicable in case of (18)'s fulfillment too.

The accumulation of experimental facts about the outer radiation belt will allow the conducting at a later date of more precise quantitative analysis of VLF-radiation. The good knowledge of this radiatiin's peculiarities should allow to forecast to a certain extent the state of the outer radiation belt.

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****** THE END ******

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